Exam 7 High-Level Summaries

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Brosius

Loss Development Using Credibility

Overview

Brosius introduces the Least Squares method for estimating loss reserves and compares this method to the traditional Chain Ladder and the Budgeted Loss (Expected Loss) methods. The key theme of this paper is that the Least Squares method is a credibility weighting of the Link Ratio (Chain Ladder) and Budgeted Loss methods.

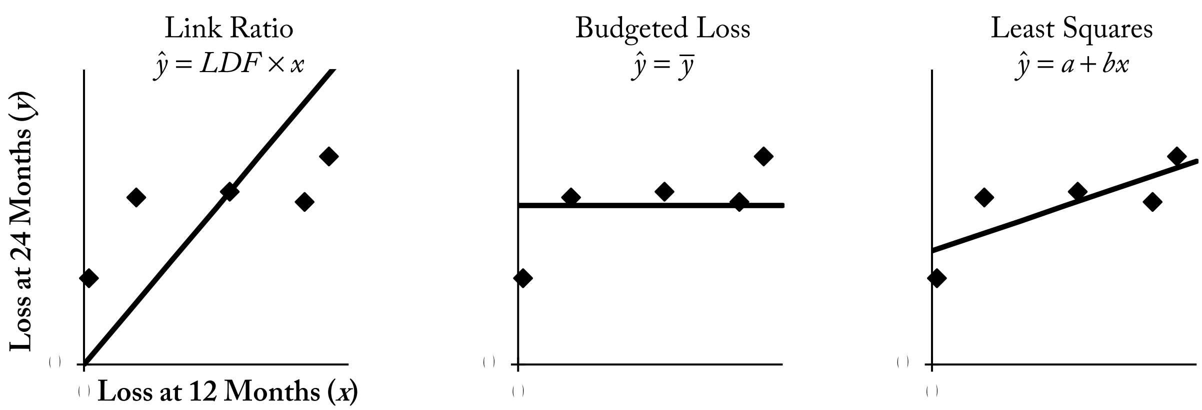
Least Squares Method

The Least Squares method fits a regression line through the data to estimate developed losses ($\hat{y}$).

$$b = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{x^2} - \overline{x}^2} \qquad a = \overline{y} - b \cdot \overline{x} \qquad \hat{y} = a + bx$$

See the "Brosius – Least Squares" recipe.

Comparison of Methods



Least Squares as Credibility Weighting

The Least Squares method is a credibility weighting of the Link Ratio and Budgeted Loss methods. The Link Ratio and Budgeted Loss methods represent the extremes:

- Link Ratio: Places 100% credibility on loss experience and 0% on expected losses
- Budgeted Loss: Places 0% credibility on loss experience and 100% on expected losses

<u>Least Squares Credibility Formula</u>

The Least Squares method is flexible and places more (or less) credibility on the loss experience as appropriate.

Below are the key formulas for calculating the credibility on the link ratio method. The factor *c* is just the LDF for the link ratio method. The credibility is the ratio of *b* to the LDF. The closer *b* is to the LDF, the higher the credibility weighting the least squares method places on the link ratio method.

$$c = LDF = \frac{\bar{y}}{\bar{x}} \qquad Z = \frac{b}{c}$$

Credibility-weighted formula:

$$\hat{y} = Z \times \frac{x}{d} + (1-Z) \times \mathrm{E}\left[y\right]$$

$$\hat{y} = Z \times LDF \times x + (1-Z) \times \mathrm{E}\left[y\right]$$

Special Cases

- If x and y are completely uncorrelated, then b = 0, resulting in the Budgeted Loss method where $\hat{y} = a$.
- If the regression line fits through the origin, then a = 0, resulting in the Chain Ladder method where $\hat{y} = bx$.
- The Bornhuetter-Ferguson method is a special case of Least Squares where b = 1. The BF method can be problematic if negative loss development is expected. The Least Squares method would allow b to adapt to the observed data.

Potential Problems (and Fixes) with Least Squares

The intercept is negative (a < 0):

- This causes the estimate of developed losses ($\hat{y}$) to be negative for small values of *x*.
- Solution: Use the link ratio method instead.

The slope is negative (b < 0):

- This causes the estimate of *y* to decrease as *x* increases
- Solution: Use the budgeted loss method instead

Key Assumptions for Least Squares

Least Squares assumes a steady distribution of random variables X and Y

- Least Squares is inappropriate if there's a systematic shift in the book of business.

Advantages of Least Squares

- Least Squares is more flexible than the link ratio, budgeted loss, and BF methods.
- Least Squares is a credibility weighting of the link ratio and budgeted loss estimates. It gives more (or less) credibility to the loss experience (*x*) as appropriate.

- Least Squares produces more reasonable results when the data has severe random, year-to-year fluctuations (e.g. a small book of business or thin data).

Adjustments to the data when using Least Squares

- When using incurred loss data, the data should be adjusted for inflation so that all accident years are on a constant-dollar basis.
- If there is significant growth in the book of business, you should divide the data by an exposure basis.

Hugh White's Question

If reported losses(x) come in higher than expected, the different methods will estimate different changes to the outstanding loss reserve:

- **Budgeted Loss Method (fixed prior case)** – The ultimate loss estimate is fixed, so we *decrease the loss reserve estimate* by the same amount as the unexpected increase in reported losses. This method treats the increased loss as losses coming in faster than expected.

- **BF Method** – The ultimate loss estimate increases by the amount losses were greater than expected. The *loss reserve is unchanged*. The BF method treats the unexpected increased loss as a random fluctuation (e.g. a large loss).

- **Link Ratio Method (fixed reporting case)** – The ultimate loss estimate increases in proportion to the excess losses by applying the LDF, so we *increase the loss reserve estimate*. This method assumes that a fixed percentage of ultimate losses is reported, so if reported losses increases, the ultimate loss estimate will increase proportionally.

Theoretical Models – Testing Least Squares

The purpose of this section is to test the least squares model against a few different theoretical loss models. With a theoretical model, we can use Bayes' Theorem to calculate the "correct" loss model and then see whether the least squares, budgeted loss or link ratio models have the same form as the Bayesian approach.

Model	Form	Model Constraints
Least Squares	$\hat{y} = a + bx$	
Link Ratio	$\hat{y} = bx$	a = 0
Budgeted Loss	$\hat{y} = a$	b = 0
Bornhuetter-Ferguson	$\hat{y} = a + x$	b = 1

Simple Model

- The number of ultimate claims incurred (Y) is either 0 or 1 with equal probability
- If there is a claim (Y = 1), there is a 50% chance it's reported by year end (X)

Using Bayes' Theorem, the best estimate of ultimate claims given x is $\hat{y} = \frac{1}{3} + \frac{2}{3}x$. This is the form $\hat{y} = a + bx$, so only the Least Squares method is compatible.

Poisson - Binomial Model

- The number of ultimate claims incurred (Y) is Poisson with mean μ
- Any given claim has probability *d* of being reported by year end

Using Bayes' Theorem, the best estimate of ultimate claims is $\hat{y} = x + \mu(1-d)$. This is the same form as both the Least Squares method and BF method, since b = 1.

Negative Binomial – Binomial Model

- The number of ultimate claims incurred (Y) is Negative Binomial with parameters (*r*,*p*)
- Any given claim has probability *d* of being reported by year end

This model also has a Baysian estimate with the same form as the Least Squares method, but the other methods will be incorrect.

Linear Approximation (Bayesian Credibility Approach)

We can only calculate the true Bayesian estimate by assuming a distribution for Y and X|Y, but that's not practical. Instead, we're going to find the best linear approximation to the Bayesian estimate of ultimate losses with Bayesian credibility, L(x).

$$L(x) = (x - \mathrm{E}[X])\frac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)} + \mathrm{E}[Y]$$

Below is how a large reported loss (increasing *x*) can change the loss reserves, corresponding with the three different answers to Hugh White's question. For $x > \mathrm{E}[X]$:

- $\mathrm{Cov}(X,Y) < \mathrm{Var}(X)$: loss reserve *decreases*
- $\mathrm{Cov}(X,Y) = \mathrm{Var}(X)$: loss reserve *unaffected* (ultimate loss increases by the increase to *x*)
- $\mathrm{Cov}(X,Y) > \mathrm{Var}(X)$: loss reserve *increases*

Using loss data, we can estimate $\mathrm{Cov}(X,Y)$, $\mathrm{Var}(X)$, and $\mathrm{E}[Y]$, which gets us right back to the Least Squares method.

$$L(x) = \hat{y} = (x - \bar{x})\frac{\overline{xy} - \bar{x}\cdot\bar{y}}{\overline{x^2} - \bar{x}^2} + \bar{y}$$

The key point is that the *least squares method is the best linear approximation to the Bayesian estimate*, although there will be sampling error in the parameter estimates of *a* and *b*.

Using simulated data from the Poisson-Binomial model, the Least Squares method fits the data better than the link ratio method and has a lower MSE.

Bayesian Credibility

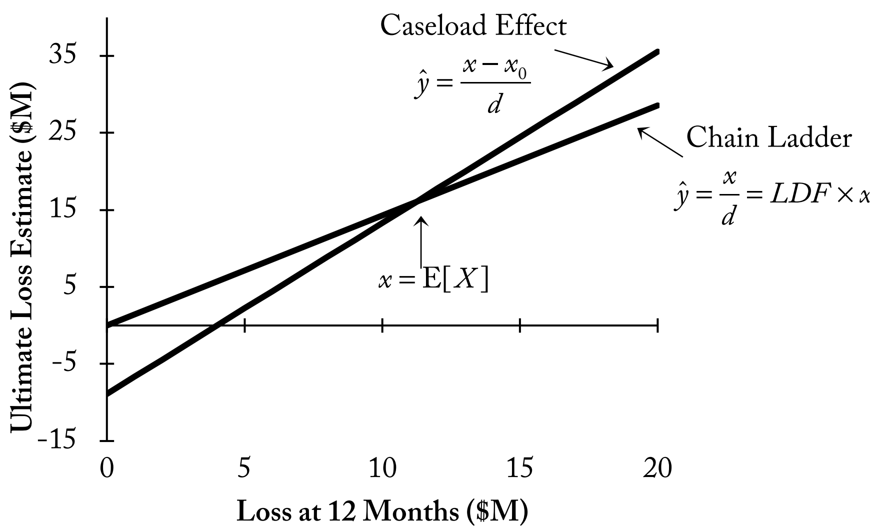
If the book of business changes significantly, we can't use the regular Least Squares method. But, if we make a few assumptions about the expected ultimate losses (Y) and the percent reported ($\frac{X}{Y}$), then we can calculate a Bayesian credibility estimate of ultimate losses (See the "Brosius – Bayesian Credibility" recipe).

Caseload Effect

The regular Bayesian credibility formula assumes that the expected percent of losses reported is the same no matter how large ultimate loss (Y) is. The caseload effect says that if ultimate loss is higher, then we would expect a lower percent of losses to be reported at time x (See the "Brosius – Caseload Effect" recipe).

Bayesian credibility still works, but the Bayesian credibility formula needs to be modified. Instead of using a fixed percent reported, the expected percent reported is lower if ultimate losses (Y) are higher. Below is a graphical view of the caseload effect and how the caseload effect estimate compares to the unmodified chain ladder estimate.

- If x > E[X], the caseload estimate will be higher than the unmodified chain ladder estimate.
- If x < E[X], the caseload estimate will be lower than the unmodified chain ladder estimate.



Recipes for Calculation Problems

- Least Squares Method
- Bayesian Credibility
- Caseload Effect

Mack (2000)

Credible Claims Reserves: The Benktander Method

Overview

Mack (2000) is a calculation-heavy paper. Most importantly, you need to be able to estimate Benktander reserves a number of different ways based on how the problem is written. Another key concept to remember is that the Benktander ultimate loss estimate is a credibility-weighting of the chain ladder and expected loss ultimates.

Benktander Method

The Benktander method can be calculated as a second iteration of the BF procedure or as a credibility-weighting of the Chain Ladder and Expected Loss ultimates (see the "Mack (2000) – Benktander Method" recipe).

Benktander as a second iteration of the BF procedure

Iteration 1 – Bornhuetter-Ferguson:

$$\boxed{U_{BF} = C_k + q_k U_0} \qquad Ult_{BF} = Loss + (1 - \%Paid) \times Prem \times ELR$$

Iteration 2 – Benktander:

$$\boxed{U_{GB} = C_k + q_k U_{BF}} \qquad U_{GB} = Loss + (1 - \%Paid) \times Ult_{BF}$$

Benktander as a credibility-weighting of the Chain Ladder and Expected Loss Ultimates

Chain Ladder Ultimate:

$$\boxed{U_{CL} = \frac{C_k}{p_k}} \qquad Ult_{CL} = Loss \times CDF$$

Benktander:

$$\boxed{q_k = 1 - \frac{1}{CDF}}$$

$$\boxed{U_{GB} = \left(1 - q_k^{\,2}\right) U_{CL} + q_k^{\,2} \times U_0} \qquad Ult_{GB} = \left[1 - \%Unpaid^2\right] \times Ult_{CL} + \%Unpaid^2 \times Prem \times ELR$$

Benktander as a credibility-weighting of the Chain Ladder and BF Reserves

$$\boxed{R_{GB} = \left(1 - q_k\right) R_{CL} + q_k \times R_{BF}} \qquad Resv_{GB} = \left[1 - \%Unpaid\right] \times Resv_{CL} + \%Unpaid \times Resv_{BF}$$

Advantages of the Benktander Method

- Outperforms the BF and Chain Ladder methods in many circumstances
- The MSE of the Benktander reserve is almost as small as that of the optimal credibility reserve

Iterated BF Method

The Benktander method is a second iteration of the BF procedure. This is how the iteration works:

1. Start with an ultimate loss estimate, $U^{(m)}$. For $U^{(0)}$, use the expected loss estimate.
2. Apply the BF procedure to get a new loss reserve estimate:

$$\boxed{R^{(m)} = q_k U^{(m)}} \qquad Resv^{(m)} = \%Unpaid \times Ult^{(m)}$$

3. Get a new ultimate loss estimate by adding the losses-to-date to the reserve. This is the starting ultimate for the next iteration:

$$\boxed{U^{(m+1)} = C_k + R^{(m)}} \qquad Ult^{(m+1)} = Loss_k + Resv^{(m)}$$

The ultimate loss estimate ($U^{(m)}$) can be rearranged as a credibility weighting of the Chain Ladder ultimate (U_{CL}) and expected loss ultimate (U_0). Also, the loss reserve estimate ($R^{(m)}$) can be rearranged as a credibility weighting of the Chain Ladder reserve (R_{CL}) and the BF reserve (R_{BF}):

$$\boxed{U^{(m)} = \left(1 - q_k^m\right) U_{CL} + q_k^m \times U_0} \qquad \boxed{R^{(m)} = \left(1 - q_k^m\right) R_{CL} + q_k^m \times R_{BF}}$$

m	Starting Ultimate ($U^{(m)}$)		New Reserve ($R^{(m)}$)
0	$U_0 = Prem \times ELR$	$\times\%Unpaid$ →	$R_{BF} = q_k U_0$
1	$U_{BF} = Loss + R_{BF}$ $U^{(1)} = \left(1 - q_k^1\right) U_{CL} + q_k^1 \times U_0$	← $+ Loss$	$R_{GB} = q_k U_{BF}$ $R^{(1)} = \left(1 - q_k^1\right) R_{CL} + q_k^1 \times R_{BF}$
2	$U_{GB} = Loss + R_{GB}$ $U^{(2)} = \left(1 - q_k^2\right) U_{CL} + q_k^2 \times U_0$		$R^{(2)} = q_k U^{(2)}$ $R^{(2)} = \left(1 - q_k^2\right) R_{CL} + q_k^2 \times R_{BF}$
⋮	⋮		⋮
∞	$U^{(\infty)} = U_{CL}$		$R^{(\infty)} = R_{CL}$

As the number of iterations increases, the weight on the chain ladder method increases until it converges to the chain ladder method entirely (as $m \to \infty$).

Recipes for Calculation Problems

- Benktander Method

Overview

The reserve estimate method in Hürlimann is a credibility-weighted method that's very similar to the Mack (2000) method. The key difference is that Hürlimann uses expected incremental loss ratios (m_k) to specify the payment pattern instead of using LDFs calculated directly from the losses.

Hürlimann uses two new reserving methods based on the loss ratio payout factors, p_i:

- Individual Loss Ratio Reserve (R^{ind}) – Similar to the chain ladder method
- Collective Loss Ratio Reserve (R^{coll}) – Similar to the cape cod method

The key idea from Hürlimann is that R^{ind} and R^{coll} represent extremes of credibility on the actual loss experience and we can calculate a credibility-weighted estimate that minimizes the MSE of the reserve estimate.

Credible Loss Ratio Claims Reserve

Individual Loss Ratio Claims Reserve (R^{ind})

$$\boxed{R^{ind} = \frac{q_i \cdot Loss_i}{p_i}} \qquad R^{ind} = \frac{\%Unpaid_{AY} \cdot Loss_{AY}}{\%Paid_{AY}}$$

R^{ind} – 100% credibility on losses-to-date

Collective Loss Ratio Claims Reserve (R^{coll})

$$\boxed{R^{coll} = q_i \cdot Prem \cdot ELR} \qquad R^{coll} = \%Unpaid_{AY} \cdot Premium_{AY} \cdot ELR$$

R^{coll} – 0% credibility on losses-to-date

Credibility-Weighted Reserve Estimate

We can calculate a new, credibility-weighted estimate, based on R^{ind} and R^{coll}, that minimizes the mean squared error (MSE) and variance of the loss reserve estimate.

$$\boxed{R_i = Z_i \cdot R_i^{ind} + (1 - Z_i) \cdot R_i^{coll}}$$

Hürlimann uses three different credibility methods:

- Benktander (also in Mack (2000))
- Neuhaus
- Optimal credibility weighting (minimizes MSE)

Method	Z_i
Benktander(Z_i^{GB})	$\boxed{Z_i^{GB} = p_i}$
Neuhaus(Z_i^{WN})	$\boxed{Z_i^{WN} = p_i \cdot ELR}$
Optimal(Z_i^{opt})	$\boxed{Z_i^{opt} = \frac{p_i}{p_i + \sqrt{p_i}}}$

Advantages over the Mack (2000) approach

- Straightforward calculation of the optimal credibility weight
- Different actuaries get the same result using the collective loss ratio claims reserve with the same premiums (BF method requires an ELR assumption)

Advantage of the optimal credibility weight reserve (R^{opt})

- Minimizes MSE and variance of the loss reserve estimate
  - Note: the MSE from Benktander and Neuhaus are close to the optimal credibility MSE

Optimal Credibility Weights

Hürlimann derives the optimal credibility weights in sections 4-6. Many of the formulas are intermediary formulas in the derivation. For the exam, I would focus primarily on the final, simplified optimal credibility weight as well as the generalized optimal credibility formula (see the "Optimal Credibility Weights" recipe).

If we assume that the variance of the ultimate loss is the same as the variance of the burning cost ultimate loss estimate, $\text{Var}(U_i) = \text{Var}(U_i^{BC})$, we get the simplified optimal credibility weight formula. If we make a different assumption, then you need to use the generalized version of the formula.

You should definitely know the simplified optimal credibility weight formula. A potential twist to a question would be to use a different assumption, such as $\text{Var}(U_i) = 2 \times \text{Var}(U_i^{BC})$, and then use the generalized formula to calculate the optimal credibility weight.

Application to Standard Approaches

Hürlimann derived the optimal credibility weight formula for the loss ratio claims reserve approach. It can also be used with a more traditional approach, using LDFs to calculate the payout pattern (p_i^{CL}). With the LDF-based payout pattern, you can then calculate the reserve estimate as a credibility-weighting of the Chain Ladder and Cape Cod (or BF) reserve estimates using the Benktander, Neuhaus or optimal credibility weights.

Credibility-Weighted Cape Cod Approach

- Use LDFs to calculate the payout pattern (p_i^{CL})
- Calculate the ELR using the Cape Cod method

Credibility-Weighted Bornhuetter Ferguson Approach

- Use LDFs to calculate the payout pattern (p_i^{CL})
- The ELR is an assumption

Recipes for Calculation Problems

- Credible Loss Ratio Claims Reserve
- Optimal Credibility Weights

Clark

LDF Curve-Fitting and Stochastic Reserving

Overview

Clark is a calculation-heavy paper and you should be able to do the Variance of Reserve calculations effortlessly by the time of the exam, a problem which comes up regularly. You should also be prepared for questions about model assumptions, diagnostic graphs and how to evaluate results for reasonableness.

The focus of Clark is to create develop a loss reserving model that estimates a central loss reserve estimate (with the LDF or Cape Cod method) and can calculate the variance of the reserve estimate.

Goals of a loss reserving model

- Mathematically describe loss emergence to estimate loss reserves
- Estimate the reserve range around the expected reserve, due to variance from:
  - Process variance - uncertainty due to randomness
  - Parameter variance - uncertainty in expected value

Expected Loss Emergence

Instead of using LDFs directly, we're going to fit a curve, *G(x)*, to incremental losses using MLE in order to get the best-fitting parameters. Then, we'll use this curve to estimate the payment pattern (from 0% to 100%) as an accident year matures.

Clark uses two different curves, the Loglogistic and Weibull curve. The Weibull curve has a lighter tail.

Loglogistic

$$G(x) = \frac{x^\omega}{x^\omega + \theta^\omega}$$

Weibull

$$G(x) = 1 - e^{-\left(\frac{x}{\theta}\right)^\omega}$$

x - *average age* of loss occurrence.

Advantages of using a parameterized curve for the loss emergence pattern

- Estimating unpaid losses is simplified (only need 2 parameters)
- Can also use data that's not from a triangle with evenly spaced evaluation dates
- Payout pattern, *G(x)*, is a smooth curve and doesn't overfit like age-to-age factors might

Advantage of using data in a tabular format

- Can use data at irregular evaluation periods or when you only don't have the full triangle

Process Variance

Process variance is the variance due to randomness in the insurance process.

Loss Model Assumptions

- Assume incremental losses have a constant variance/mean ratio, σ^2
- Assume incremental losses follow an over-dispersed Poisson model

Fitting the loss emergence curve

We find the best-fitting curve using the maximum likelihood method. For a given set of parameters, we calculate the expected incremental losses, μ_i, for each cell of the loss triangle. Then, we compare the likelihood that the actual incremental losses came from an over-dispersed Poisson distribution with the parameters μ_i and σ^2.

The goal is to find the set of parameters that maximize the log-likelihood function. See the "Finding Best-Fit Parameters with MLE" recipe for an example of how to do this.

Advantages of using an over-dispersed Poisson distribution to model incremental losses:

- Using a scaling factor, σ^2, allows us to match the 1st and 2nd moments of other distributions
- The Maximum Likelihood Estimate reproduces LDF and Cape Cod loss reserve estimates

Parameter Variance

Parameter variance is the variance in the estimate of the parameters. It's calculated based on the Rao-Cramer lower-bound approximation, using the second derivative information matrix. The information matrix is used to calculate the covariance matrix, which is used to calculate the parameter variance. The actual calculation of the parameter variance is too complex for the exam.

Key Model Assumptions

- **Incremental losses are iid**
  - Independent – One period doesn't impact surrounding periods (this assumption fails if there are calendar year effects such as inflation)
  - Identically distributed – Assume the same emergence pattern, G(x), for all accident years (this assumption fails if the mix of business or claims handling changes)
- **Variance/Mean Scale parameter, σ^2, is fixed and known**
  - We ignore the variance of σ^2
- **Variance estimates use approximation to the Rao-Cramer lower bound**

The model assumptions mean there's a potential that future losses have higher variance than what the model indicates.

LDF Method

A problem with the LDF method is that there is a parameter for each accident year for the current $Loss_{AY}$. See the "Variance of Reserves (LDF Method)" recipe for how to calculate reserve variance.

Cape Cod Method

The Cape Cod method uses additional information, an exposure base. Clark recommends on-level earned premium, but another exposure base can be used as long as it's proportional to ultimate expected losses by accident year.

Premium must be on-leveled so that we can assume a constant ELR across all accident years. We could also adjust for loss trend net of exposure trend so that all accident years are at the same cost level.

LDF Method vs. Cape Cod Method

- LDF method over-parameterizes the model, fits to the "noise" in the data
  - For a 10-yr triangle, there are 12 parameters to estimate and only 55 data points
- Cape Cod has lower parameter variance because there are fewer parameters to estimate and it uses more information (on-level premium)

Variance of Loss Reserves

Once you have the loss reserve estimate, the scale parameter and the parameter variance, you can calculate the variance around the loss reserve estimate.

$$Process\,Var = \sigma^2 \cdot Resv$$

$$Total\,Variance = Process\,Variance + Parameter\,Variance$$

Diagnostics

ELR for Cape Cod

The Cape Cod method assumes a constant ELR across all accident years. Test this assumption by graphing the estimated ultimate loss ratios by accident year.

The estimated ultimate loss ratios should be random around the ELR with no patterns or trends. If there is a pattern, then the assumption of a constant ELR isn't reasonable and the Cape Cod reserve estimate could be biased.

$$\widehat{Ult\,Loss\,Ratio}_{AY} = \frac{Loss_{AY}}{Prem_{AY} \times G(x)}$$

Residual Graphs

$$r_{AY,k} = \frac{IncLoss_{AY,k} - \mu_{AY,k}}{\sqrt{\sigma^2 \cdot \mu_{AY,k}}}$$

$$Norm.residual = \frac{actual\ incremental - expected\ incremental}{\sqrt{\sigma^2 \cdot expected\ incremental}}$$

Residual graphs are an important diagnostic to test the underlying assumptions. Below are the graphs Clark specifically mentions and what assumptions they test:

- **Normalized Residuals vs. Increment Age**
  - Tests how well the loss emergence curve fits incremental losses at different development periods.
- **Normalized Residuals vs. Expected Incremental Loss (μ_i)**
  - Tests the variance/mean ratio σ^2: If the variance/mean ratio is ***not*** constant, we should see residuals clustered closer to zero at either high or low expected incremental losses.
- **Normalized Residuals vs. Calendar Year**
  - Tests whether there are diagonal effects (e.g. high inflation in a calendar year)

For all the graphs, the residuals should be random around zero with no patterns or autocorrelations. If this isn't the case, then some assumptions of the model are incorrect.

Process vs. Parameter Variance

We should see that parameter variance is greater than process variance. This is because most of the uncertainty is due to the inability to estimate the expected reserve (parameter variance) rather than uncertainty due to random events (process variance).

The reason for this is that there are so few data points in a loss triangle to estimate the parameters. The Cape Cod method lowers the parameter variance by including the exposure data.

Other Calculations

Variance of Prospective Losses

The regular Clark method calculates reserve variance for past accident years. This same approach can be used to calculate variability around prospective losses for the prospective accident year (period). See the "Variance of Prospective Losses" recipe for this calculation.

Calendar Year Development

Instead of estimating the total unpaid loss reserve, we can calculate the unpaid losses that we expect will be paid over the next calendar year and create a range around that.

The advantage of this type of calculation is that the model can be tested in a relatively short period of time. After one year, the actual 12-month loss development can be compared to the original forecasted range to test whether the actual development falls within the range. See the "Variance of Calendar Year Development" recipe for this calculation.

Variance of Discounted Reserves

The discounted paid loss reserve is calculated by discounting the future payments at the half-year mark. For the exam a calculation problem doesn't seem testable.

One key point is that the CV of the discounted loss reserve is smaller than the CV of the undiscounted loss reserve. This is because the tail of the payout curve has the greatest parameter variance, but is discounted the most.

Adjustments for Other Exposure Periods

The formula for *G(x)* is only valid on an accident year basis where the first development period is at 12 months. If the first development period is shorter than 12 months or policy year is used, then you need to make some adjustments.

Percent of ultimate loss as of time *t*, with annualization:

$$G(t) = expos(t) \cdot G(x) \quad \text{where} \quad x = AvgAge(t)$$

	Accident Year	Policy Year
expos(t)	$= \begin{cases} \frac{t}{12} & t \le 12 \\ 1 & t > 12 \end{cases}$	$= \begin{cases} \frac{1}{2} \cdot \left(\frac{t}{12}\right)^2 & t \le 12, \text{ note: } expos(12) = 50\% \\ 1 - \frac{1}{2}\left(2 - \frac{t}{12}\right)^2 & 12 < t \le 24 \\ 1 & t > 24 \end{cases}$
AvgAge(t)	$= \begin{cases} \frac{t}{2} & t \le 12 \\ t - 6 & t > 12 \end{cases}$	$= \begin{cases} \frac{t}{3} & t \le 12 \\ \frac{(t-12) + \frac{1}{3}(24-t)(1-expos(t))}{expos(t)} & 12 < t \le 24 \\ t - 12 & t > 24 \end{cases}$

I would know all the formulas for accident year. For policy year, I would make sure to know the special cases where t = 12 months and where t = 24:

Policy Year	*expos(t)*	*AvgAge(t)*
t = 12	50%	4
t = 24	100%	12

Recipes for Calculation Problems

- Variance of Reserves (LDF Method)
- Variance of Reserves (Cape Cod Method)
- Normalized Residuals
- Variance of Prospective Losses
- Variance of Calendar Year Development
- Finding Best-Fit Parameters with MLE

Mack (1994)

Measuring the Variability of Chain Ladder Reserve Estimates

Key Ideas

The goal of this paper is to use the chain ladder method to create a confidence interval around the ultimate loss and estimated loss reserve.

Mack (1994) focuses on two main ideas:

1. **Three main assumptions** underly the chain ladder method. For a loss triangle, we should look at different diagnostic graphs and tests to see whether the chain ladder method is appropriate or not.
2. Based on the three assumptions, we can calculate the estimated loss reserve and the standard error of the estimated loss reserve for each accident year and for all years combined with the chain ladder method. Then, after making an assumption about the distribution of the loss reserve (e.g. lognormal), we can calculate loss reserve confidence intervals.

Mack Assumptions

1. Expected losses in the next development period are proportional to losses-to-date.

$$\mathrm{E}\left[C_{i,k+1} \mid C_{i,1},\cdots,C_{i,k}\right] = C_{i,k} \cdot LDF$$

- The chain ladder method uses the same LDF for each accident year
- Uses most recent losses-to-date to project losses, ignoring losses as of earlier development periods

2. Losses are independent between accident years.

3. Variance of losses in the next development period is proportional to losses-to-date with proportionality constant α_k^2 that varies by age.

$$\mathrm{Var}\left[C_{i,k+1} \mid C_{i,1},\cdots,C_{i,k}\right] = C_{i,k} \cdot \alpha_k^2$$

Alternative Variance Assumptions (Weighted LDFs)

The chain ladder method in Mack (1994) uses volume-weighted LDFs. This implies that variance of losses in the next development period (*$Loss_{k+1}$*) is proportional to losses-to-date (*$Loss_k$*), Assumption 3.

If we calculate the LDFs a different way, such as the simple average of the age-to-age factors or age-to-age factors weighted by $Loss_k^2$, then we're making a different variance assumption.

Variance Assumptions

$\mathrm{Var}(C_{k+1}) \propto$	LDF calc. Weight	LDF
1	$C_{i,k}^2$	$C_{i,k}^2$ -wtd
$C_{i,k}$	$C_{i,k}$	Vol-weighted
$C_{i,k}^2$	1	Simple Avg

Variance of Reserves

Using the three chain ladder assumptions, we can calculate the variance of the reserve estimate with Mack's formula for the standard error of the reserve. We get the estimated reserve and the standard error of the estimate by accident year and for the overall reserve.

One disadvantage of the Mack method (compared to the bootstrap method) is that it doesn't tell us about the shape of the loss reserve distribution. It only gives us the mean and standard deviation. To create confidence intervals, we need to make an assumption about the distribution.

Reserve Confidence Intervals

Once we've done all the calculations, we have an expected loss reserve estimate (the loss reserve estimate from the standard chain ladder method) and the standard error (standard deviation) of the loss reserve estimate. Besides these values, the method doesn't tell us anything else about the distribution of the loss reserve estimate.

So, we're going to assume a distribution and use the mean and standard deviation of the loss reserve that we calculated. Mack looks at two distributions: Normal and Lognormal.

Normal Distribution

$$\text{C.I.} = R \pm z \cdot \text{s.e.}(R)$$

- If min of C.I. is negative OR s.e.(R) > 50% of R, use Lognormal

Lognormal Distribution

$$\sigma^2 = \ln\left[1 + \left(\frac{\text{s.e.}(R)}{R}\right)^2\right]$$

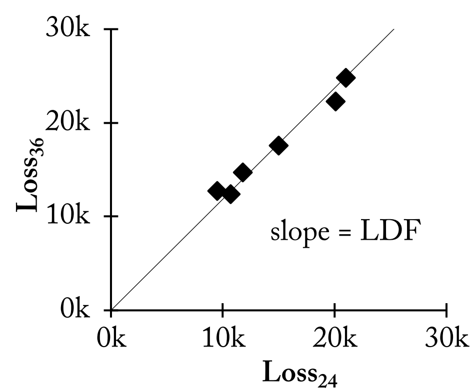
$$\text{C.I.} = R \cdot \exp\left[\pm z \cdot \sigma - \frac{\sigma^2}{2}\right]$$

Checking the Chain Ladder Assumptions

The three Mack assumptions have significant implications, so we should take a look at some different tests and diagnostics to see how well the assumptions hold up.

Plot of Cumulative Losses from Adjacent Periods

This plot tests assumption 1. We want to see if losses at the next period are proportional to losses-to-date with no intercept.



If the assumption holds, you should see:

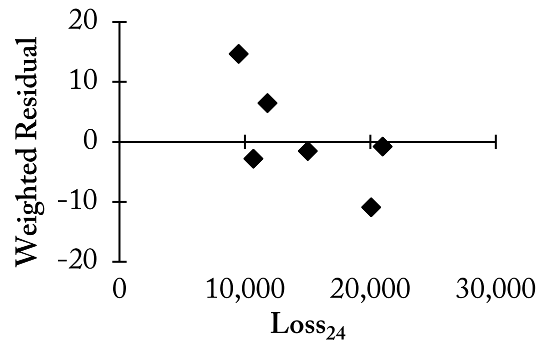
- Linear relationship between $Loss_{k+1}$ and $Loss_k$ through the origin (no intercept)
- Line should go through the data points

If the assumption is violated, you might see:

- The data points show that there should be an intercept term in the regression line
- The relationship isn't linear

Plot of Weighted Residuals

This plot primarily tests assumption 3, the variance assumption.



If the assumption holds, you should see:

- Residuals should be random around zero with no significant trends or patterns.

If the assumption is violated, you might see:

- Variance of residuals is higher at one end of the graph than at the other
- Residuals show an increasing (or decreasing trend)

If the residuals for a few development periods aren't random, we can graph the residual plots using the LDFs for the alternative variance assumptions: $\text{Var}(C_{k+1}) \propto C_k^2$ and $\text{Var}(C_{k+1}) \propto 1$. If the residual plots for one of the alternative assumptions is more random, then we could replace the volume-weighted LDF with the alternative LDF (f_{k0} or f_{k2}).

Testing Assumption 2 – Independence between accident years (Calendar Year Test)

The assumption of independence between accident years implies that there are no calendar year effects that impact losses from multiple accident years (causing a dependence between accident years).

We test the null hypothesis that there are no calendar year effects with Mack's calendar year test (see the "Mack (1994) – Calendar Year Test" recipe). If there are significant calendar year effects, then assumption 2 is violated.

Examples of calendar year effects:

- Major changes in claims handling practices
- Major changes in setting case reserves
- Unexpectedly high (or low) inflation
- Significant changes due to court decisions

Testing for Correlation Between Adjacent LDFs (Assumption 1)

The first assumption implies that subsequent development factors are uncorrelated (e.g. high development from ages 12 to 24 gives no information about how losses develop from age 24 to 36).

In the chain ladder method, we always use the same LDF from ages 24 to 36 no matter how losses developed from ages 12 to 24. For instance, if the overall LDF from ages 12 to 24 is 3.5 and a particular AY developed from 1,000 to 5,500 (age-to-age factor of 5.5), we still expect the same % development from 24 to 36.

- **Example:** If a book of business typically shows a smaller-than-average increase, $Loss_{k+1}/Loss_k < LDF_k$, after a larger-than-average increase, $Loss_k/Loss_{k-1} > LDF_{k-1}$, then the chain ladder method would ***not*** be appropriate.

Mack (1994) uses Spearman's rank correlation to test for correlation between LDFs (See the "Mack (1994) – Correlation of Adjacent LDFs" recipe).

The Mack Correlation of Adjacent LDFs test looks at the triangle as a whole, not at each column-pair separately. This is because it's more important to know if correlation between LDFs prevails throughout the triangle.

Advantages of using Spearman's rank:

- The test is distribution-free (doesn't assume LDFs are from a normal distribution)
- Differences in variances of LDFs between development periods is less important because it uses ranks

Reviewing Results for Reasonableness

- MSE of the total reserve is greater than the sum of the MSE from the reserve for individual accident years.
    - Because the same LDFs are used for all accident years, the loss reserve estimates are positively correlated between accident years, increasing the total MSE.
- Higher standard error percentage for the oldest accident years
    - The absolute standard error should be lowest for the oldest accident years, but since the size of the loss reserve is so small, the standard error percentage will be higher.
- Higher standard error percentage for the most recent accident year (or two)
    - Uncertainty in forecasting future losses is highest for the most recent accident years because losses are immature, so the standard error percentage will be higher.

Weaknesses of the Chain Ladder method

- Estimators of the last 2 or 3 LDFs rely on very few observations
- Doesn't work well for the most recent accident year where losses-to-date provide a very uncertain base to project ultimate losses
    - Another method, such as the least squares method would put less credibility on the immature losses.

Recipes for Calculation Problems

- Residual Test
- Calendar Year Test
- Reserve Confidence Interval
- MSE Calculation
- Correlation of Adjacent LDFs
- Overall MSE Calculation

Venter Factors

Testing the Assumptions of Age-to-Age Factors

Overview

Think of Venter Factors as an extension of the Mack (1994) paper. Venter takes a look at the three key Mack assumptions underlying the chain ladder method and identifies six ways to test those assumptions. If those tests fail for a loss triangle, Venter points out a few alternative loss reserving methods to use instead.

As you go through Venter Factors, think about how it ties together with Mack (1994) to give a more complete picture of the chain ladder method and how to use the chain ladder method to create reserve ranges.

Mack Assumptions – Restated

If the assumptions below hold, then the chain ladder method gives the minimum variance unbiased linear estimator of future loss emergence.

Assumption 1

Expected incremental loss emergence is proportional to cumulative losses-to-date

$$\mathrm{E}\left[IncLoss_{AY,k+1} \mid data\right] = \left(LDF_k - 1\right) \cdot Loss_{AY,k}$$

Assumption 2

Losses are independent between accident years

Assumption 3

Variance of the next incremental loss is a function of age and cumulative losses-to-date

$$\mathrm{Var}\left[IncLoss_{AY,k+1} \mid data\right] = function\left(k, Loss_{AY,k}\right)$$

We can't directly test these assumptions, but there are six testable implications that fall out of these assumptions that we can test. If the tests for any of the implications fails, then the three Mack assumptions don't hold up, meaning that the chain ladder method doesn't give the minimum variance unbiased linear estimate of future losses.

Testable Implication 1 – Significance of development factors

- **<u>Tests Assumption 1</u>:** Linear loss emergence proportional to loss-to-date

Assumption 1 assumes incremental losses are proportional to losses-to-date with a development factor. If that's the case, the development factor ***should*** be statistically significant.

Later in the paper, Venter shows the regression including a constant, $\hat{y} = a + bx$. This tests the alternative linear with constant (least squares) model compared to the chain ladder model and also indirectly tests for implication 1. Excluding the constant term in the regression will give a more direct test of implication 1.

One last point: You can't just do a regression of $Loss_{k+1}$ vs. $Loss_k$. This is because what we really want to predict is the *incremental* loss from one period to the next. If we use $Loss_{k+1}$, then comparing the coefficient b to the standard deviation directly will overstate the significance of the factor.

Testable Implication 2 – Superiority to alternative loss emergence patterns

- **<u>Tests Assumption 1</u>:** Linear loss emergence proportional to loss-to-date

If assumption 1 is correct, then the chain ladder model should predict incremental losses better than alternative loss emergence patterns. We can use a goodness-of-fit test to compare the alternative models to the chain ladder method.

Different Loss Emergence Patterns

Expected Incremental Loss is…	Model
…proportional to loss-to-date	Chain Ladder
…proportional to loss-to-date with a constant	Least Squares
…percentage of ultimate loss	Bornhuetter-Ferguson: f(d)h(w)

Alternative Pattern 1 – Linear with a constant (least squares method)

$$E\left[IncLoss_d\right] = \underset{\text{factor}}{f(d)} \cdot Loss_k + \underset{\text{constant}}{g(d)}$$

Including a constant term is often significant in the first development period or two, especially for highly variable and long-tail lines.

Test this alternative pattern by running a linear regression of $IncLoss_{k+1}$ vs. $Loss_k$ to get the factors of the development formula $\hat{y} = a + bx$. If the constant is statistically significant, this loss emergence pattern is more strongly supported than the chain ladder method.

Alternative Pattern 2 – Factor times parameter (parameterized BF model)

If expected incremental loss is modeled as a percent of ultimate, we can use the parameterized BF model:

$$\boxed{E\left[IncLoss_d\right] = f(d)h(w)}$$

We need to solve for the f(d) and h(w) factors iteratively. Note that the final f(d) factors won't necessarily sum to 1. Likewise, the h(w) factors aren't equivalent to estimated ultimate loss. Instead, f(d) and h(w) are used together to estimate future incremental losses.

Compare the parameterized BF model to the chain ladder model by doing a goodness-of-fit test. If the parameterized BF model has a lower test statistic, then the loss emergence in the loss triangle is better described by the parameterized BF model.

The parameterized BF model has twice as many parameters (2n-2) as the chain ladder model. So, it's often over-parameterized, resulting in a higher adjusted SSE. We can improve the performance by reducing the number of parameters (which will increase SSE, but hopefully decrease Adjusted SSE).

The parameterized Cape Cod method uses one *h* parameter for all accident years. If the ultimate loss for each accident year is roughly the same, then this model might be an improvement over the fully parameterized BF.

This is a bit different than the normal Cape Cod model that uses the same expected loss ratio for all accident years. A possible improvement is to use a loss ratio triangle with the Cape Cod method, because a constant ultimate loss ratio over all accident years is more reasonable.

Disadvantages of the Cape Cod method:

- Requires relatively stable level of loss exposure over accident years
- Need to adjust for known exposure and price level differences between accident years

Testable Implication 3 – Linearity of the model: Residuals vs. Loss_k

- **<u>Tests Assumption 1</u>:** Linear loss emergence proportional to loss-to-date

Assumption 1 assumes incremental loss emergence is linearly proportional to loss-to-date.

If residuals show a non-linear pattern (e.g. positive – negative – positive), then incremental loss emergence is a non-linear function of loss-to-date.

Testable Implication 4 – Stability of development factors: Residuals vs. time

- **<u>Tests Assumption 1</u>:** Linear loss emergence proportional to loss-to-date

Assumption 1 uses the same development factor for all accident years, so we would expect that the appropriate development factor is stable over time.

A pattern of high/low residuals when plotted against time indicates instability and is evidence against using the same development factor for all AYs.

If the age-to-age factors appear stable, we should use all accident years when calculating the LDFs (to minimize variance). But, if it looks like the average level of the age-to-age factors is shifting over time, we could use a weighted average of the LDFs over the last n-years (e.g. 5-year weighted average LDFs).

Testable Implication 5 – Correlation of development factors

- <u>**Tests Assumption 2**</u>**:** Independence between Accident Years *

If there is significant correlation between development factors at different ages, then this is evidence against the chain ladder method.

***Confliction with Mack (1994):**
There's some overlap between Mack assumptions 1 and 2 as they're discussed in the two papers:

- In Mack (1994) (pg. 109), Mack states that the linearity assumption (assumption 1) implies that "subsequent development factors... are uncorrelated."
- In Venter Factors (pg. 832-833), Venter says that the correlation test is to check for dependencies in the triangle, which is a test on the independence assumption (assumption 2).

Just note that there's some disagreement between Venter and Mack regarding the assumption tested by correlation of development columns.

Testable Implication 6 – Significantly high or low diagonals (calendar year effects)

- <u>**Tests Assumption 2**</u>**:** Independence between Accident Years

Run a regression of incremental losses against cumulative losses at the prior development periods and include a dummy variable for each diagonal.

If any of the dummy variables are statistically significant, then this indicates calendar year effects.

Recipes for Calculation Problems

- Correlation of Development Factors
- Parameterized BF: *f(d)h(w)* – Constant Variance
- Parameterized BF: *f(d)h(w)* – Var $\propto$ *f(d)h(w)*
- Testing for Significantly High/Low Diagonals
- Goodness of Fit Test

Siewert

A Model for Reserving Workers Compensation High Deductibles

Overview

The typical loss reserve methods are appropriate for ground-up losses. Siewert focuses on different methods that can be used to estimate unpaid loss reserves for different layers of losses, such as limited losses or losses above a high deductible. One of the main challenges Siewert addresses is how to calculate LDFs that are consistent between unlimited, limited and excess layers.

Excess Loss Reserving Methods

Loss Ratio Method

Siewert applies the loss ratio method by account to reflect differences in account characteristics. There are two components to the expected excess losses:

- **Deductible Loss Charge –** Expected losses above the per-occurrence limit
- **Aggregate Loss Charge –** Expected losses in the deductible layer (below the per-occurrence limit) that are above the aggregate limit

Advantages:

- Useful for immature accident years when loss data is thin
- Loss ratio estimates can be consistent with pricing

Disadvantages:

- Ignores actual loss experience, not that useful after several years development
- May not reflect account characteristics properly if development emerges differently due to written exposures

Implied Development

The ultimate excess loss estimate is the difference between the unlimited and limited ultimate loss estimates (calculated using the unlimited and limited losses and LDFs).

$$\boxed{Ult^{excess} = Ult^{unlimited} - Ult^{limited}}$$

The limited LDFs used to calculate $Ult^{limited}$ need to reflect indexed limits adjusted for inflation over time. This is because using a constant deductible for all accident years implies increasing excess losses.

Advantages:

- Can estimate excess loss for early maturities, even if no excess losses have emerged yet
- Limited LDFs are more stable than excess LDFs used for direct development

Disadvantages:

- Misplaced focus – we would like to explicitly recognize excess loss development

Direct Development

Apply the XSLDF to excess loss to estimate ultimate excess loss. The XSLDF should be consistent with the limited and unlimited LDFs.

Advantages:

- Focuses explicitly on excess loss development

Disadvantages:

- XSLDFs are often highly leveraged and volatile, especially for immature periods
- Direct development doesn't produce an estimate if no excess loss has emerged yet

Credibility Weighting (Bornhuetter-Ferguson)

Excess ultimate loss estimate is a credibility weighting of the actual experience and the expected losses (loss ratio method).

Advantages:

- Can tie to the pricing estimate for immature periods when losses haven't emerged
- More stable estimate over time

Disadvantages:

- Ignores actual loss experience to the extent of the complement of credibility

Development Model

The first way we can get LDFs for different limits is to calculate them directly using full coverage loss experience. This way, we can directly create loss triangles with losses limited at different limits.

Indexed Limits

Before calculating the limited LDFs from a limited loss triangle, we need to index the limits for prior accident years. This is important so that the proportion of limited-to-excess loss around the limit is consistent from year to year.

To do this, Siewert fits an exponential curve to loss severity by accident year to calculate the annual severity trend. Then, he uses the selected severity trend to trend the loss limit backwards. Once we have the indexed limits, we can create a loss triangle with losses limited at the appropriate indexed limits and calculate the limited LDFs.

See the example below about how to calculate indexed limits:

Example 1

Given:

- Annual loss severity trend is 10%
- Deductible for an account is $250,000

Calculate the indexed limits to use for a five-year limited loss triangle.

Solution

First, set the limit for the most recent accident year to the stated limit, $250k. Then, trend the limits backward for each prior accident year.

Accident Year	Indexed Limit
2012	170,753
2013	187,828
2014	206,612
2015	227,272
2016	250,000

$227{,}272 = \frac{250{,}000}{1.10}$

Tail Factors

Siewert uses the inverse power curve to get consistent tail factors by limit. First, he fits an inverse power curve to the unlimited LDFs and select an appropriate stopping point beyond which there's no further development (e.g. 40 years). He selects the stopping point so that the inverse power curve tail factor is consistent with a tail factor based on an extended loss triangle.

To get the tail factors for the other limits, he fits the inverse power curve to the limited LDFs at each limit up to the stopping point and compound the age-to-age factors from the fitted curves.

Relationships between unlimited/limited/excess LDFs and severity relativities

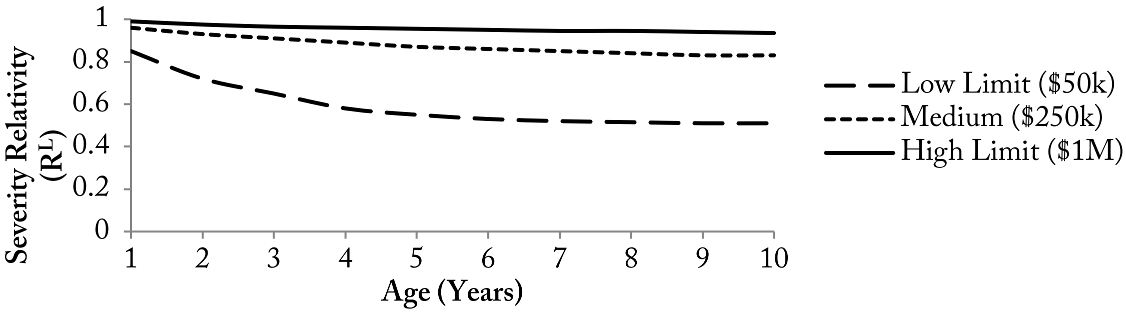
There is a relationship between limited/excess LDFs and unlimited LDFs using loss severity relativities:

$$R_t^L = \frac{\textit{Severity Limited to limit L at age t}}{\textit{Unlimited Severity at age t}}$$

Severity relativities should have the following relationships:

- **Severity relativity should decrease as age increases**
  - This is because more losses are capped at the per-occurrence limit as age increases
- **Severity relativity should be higher for a larger limit**
  - This is because a higher limit means less of the loss is capped, so the relativity is higher

You can see these relationships in the following graph of severity relativities by age and limit:



Below are the key relationships:

$$XSLDF_{t_1-t_2}^{L} = LDF_{t_1-t_2} \cdot \frac{\left(1-R_{t_2}^{L}\right)}{\left(1-R_{t_1}^{L}\right)}$$

$$LDF_{t_1-t_2}^{L} = LDF_{t_1-t_2} \cdot \frac{R_{t_2}^{L}}{R_{t_1}^{L}}$$

The unlimited LDF is a weighted average of the limited and excess LDFs. With this relationship, we can split the unlimited loss development consistently between development below and excess the limit.

$$LDF_{t_1-t_2} = R_{t_1}^{L} \cdot LDF_{t_1-t_2}^{L} + \left(1-R_{t_1}^{L}\right) \cdot XSLDF_{t_1-t_2}^{L}$$

By using the severity relativities alone to adjust the LDF, we implicitly assume no further claim count development.

Distribution Model

If we use the loss data directly, we'll find instances where limited development sometimes is greater than the unlimited development. This will happen if the limited severity relativity increases from one age to the next (remember, the severity relativity should *decrease* as age increases).

Instead of using the data directly, we can fit a separate severity distribution to losses at each age (the severity distribution for losses at 12 months will be different than the ultimate severity distribution). Then, we can calculate the severity relativities at each age for any limit we're interested in.

Siewert calculates the unlimited LDFs directly from the fitted severity relativities. Then, using the fitted severity relativities and these unlimited LDFs, we can then calculate consistent LDFs at any other limit.

Fitting the Model

Siewert uses a Weibull distribution and estimates the parameter by minimizing the chi-square error between the actual and expected severity relativities. Also, Siewert constrains the model so that the unlimited expected severity is equal to the actual unlimited severity at that development age.

Claim Count Development Assumption

In the development model, we started with selected unlimited LDFs based on unlimited loss data. A key difference here is that we're calculating the LDFs directly from the *modeled severities*. By using the severity relativities alone to calculate the LDFs, we implicitly assume no further claim count development. Because of this, an important assumption is that there's no further claim count development.

In Siewert (see pg. 232), he discusses how we can use the modeled severities to calculate the LDFs, but specifies that if there is claim count development for the earlier maturities, it needs to be reflected.

Partitioning Loss Development above and below the deductible

Because the unlimited LDF is a credibility weighting of the limited and excess LDFs, we can partition expected future development between development above and below the deductible.

As age increases, a larger portion of the expected development will be due to excess development above the limit.

$$\%Unpaid = 1 - \frac{1}{LDF} = \frac{LDF-1}{LDF}$$

$$= \frac{R_t^L \cdot \left(LDF^L - 1\right) + \left(1 - R_t^L\right) \cdot \left(XSLDF^L - 1\right)}{LDF} = \underbrace{\frac{R_t^L \cdot \left(LDF^L - 1\right)}{LDF}}_{\text{Development } \underline{\textit{below}} \text{ the deductible}} + \underbrace{\frac{\left(1 - R_t^L\right) \cdot \left(XSLDF^L - 1\right)}{LDF}}_{\text{Development } \underline{\textit{above}} \text{ the deductible}}$$

Example 2

At 48 months development, below are the paid LDFs around the 250k deductible:

Unlimited LDF	Limited LDF	XSLDF
2.152	1.959	3.148

- The 250k severity relativity at 48 months is 0.838

Partition the expected future paid development above and below the 250k deductible.

Solution

$$\%Unpaid = 1 - \frac{1}{2.152} = 53.5\%$$

$$= \frac{.838 \cdot (1.959 - 1)}{2.152} + \frac{(1 - .838) \cdot (3.148 - 1)}{2.152} = 37.3\% + 16.2\%$$

The expected development of 53.5% is made up of 37.3% of development below the deductible and 16.2% of development above the deductible. Put another way, 69.7% (=.373/.535) of expected future development is below the deductible and 30.3% is above the deductible.

Aggregate Excess of Loss Estimate

An aggregate limit caps losses in the deductible layer that the insured is responsible for. We can't calculate development factors for losses excess the aggregate limit because the data needed to do this is thin and not likely to be credible.

Instead, Siewert recommends using a collective risk model or Table M to estimate liabilities under an aggregate limit.